FSI of rocket nozzles – On the influence of simplified modeling of structural boundary conditions for an FSI experiment & scalable solvers for strongly coupled problems

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The influence of boundary conditions (BC) of the structure in a complex FSI experiment are investigated and different simple models for the BC are presented to accurately capture the behavior of the experiment. The interaction of a shock wave with a supersonic boundary layer on an elastic panel was investigated experimentally in order to validate the numerical tools developed within the SFB/TRR 40. During initial numerical analysis of the fluid-structure interaction (FSI), the support of the panel was assumed to be fully fixed which turned out to be inappropriate. As the focus of the investigations is on the FSI, it is not desirable to consider the panel clamping region with its rivets where a complex contact and prestressing scenario occurs. Hence, an elegant modeling approach is developed which accounts for the relevant aspects of the clamping while reducing its complexity by far. Translational and rotational springs are included to surrogate the “soft” behavior observed in both in static and dynamic experiments. Additionally, the compressibility of the air enclosed in the cavity below the panel turned out to influence the eigenfrequency of the panel yielding a Robin-type boundary condition.

Besides the modeling of boundary conditions, a second aspect in this work is the improvement of the linear solver for monolithic coupled multi-field problems as they occur in thermo-structure interaction. Different preconditioning strategies for the linear systems are applied to the test case which consists of the temperature dependent deformation of a rocket nozzle. Good scalability can be achieved using the developed preconditioning strategies.

1. Introduction

In order to obtain a better insight into rocket nozzles and the interaction between fluid and structure and thermal aspects therein, experimental and numerical investigations are performed within project D4 and D6 of the SFB/TRR 40. Several subproblems can be identified to reduce the complexity of the overall thermo-fluid-structure interaction problem. One setup considers the interaction of an oblique shock with a turbulent boundary layer on an elastic panel, i.e. a fluid-structure interaction problem. In [1], the turbulent fluid flow was analyzed in detail and a comparison for the fluid-structure interaction (FSI)
between experimental and numerical results was presented. In the experimental setup shown in Fig. 1(a), a wedge is pitched in order to generate a time-dependent load on the elastic panel. Hence, the impingement point of the shock on the elastic panel moves from the end point of the elastic panel onto the deformable part. Finally, the rotation of the wedge is stopped and the panel starts an oscillatory motion while loaded with the incident shock. The baseline setup contained a rigid plate instead of the elastic panel such that no FSI occurred for validation of the flow field. The fluid flow was found to be described very accurately with the Large-Eddy simulation (LES) approach proposed in [1]. However, when the elastic panel was used, significant deviations between numerical and experimental results occurred. The frequency of the panel revealed a lower value in the numerical results $f_{\text{num}} = 186$ Hz than in the experimental result $f_{\text{exp}} = 222$ Hz. Several possible reasons were given in [1] to explain the deviations. The compressible air enclosed in the cavity under the elastic panel was attributed influence on the frequency of the oscillatory panel motion. An increased damping in the experiment was explained with a possible effect of the sealing between the panel and the frame it is mounted. Furthermore, aerodynamic damping was mentioned to play a role.

In addition to investigating the effects from the boundary conditions on the panel in the FSI setup, we have developed a computational framework for the solution of large systems of linear equations arising from coupled problems. This is relevant for the second subproblem, a thermo-structure interaction problem. The framework is composed by particular solvers sharing a common feature: they can be implemented for a generic coupled problem, and therefore they can be used in a variety of applications, from thermo-mechanical models for rocket nozzles, to other interesting multiphysics problems. In particular, the computational framework has been successfully applied to different fields such as fluid-structure interaction for blood flow modeling and patient-specific pulmonary mechanics.

The proposed linear solver framework is motivated by the following fact. Most solvers for strongly coupled problems consider complex techniques designed for specific applications. This approach has the advantage that the methods can be optimized for the particular problem type leading to efficient solvers, but on the other hand, problem-specific implementations have to be developed and maintained separately. This is especially an issue in “general purpose” multiphysics codes designed to simulate different kinds of phenomena. For this reason, an alternative approach is explored. Solvers are selected that are able to be implemented for a generic coupled problem, involving an arbitrary number of fields. Then, we have implemented these methods in a unified computational framework within the high performance multiphysics code BACI. The purpose is to show that such a framework is realizable and that the selected methods are efficient and able to cope with a wide range of applications of which the thermo-mechanical system of a rocket nozzle is chosen in this contribution. A small summary and an outlook will conclude the contribution.

2. Influence of boundary conditions of the structure in a complex FSI setup

The simplified modeling of boundary conditions of the structure in an FSI setup of an oblique shock with a turbulent boundary layer on an elastic panel is investigated first. The solutions of three different numerical tools are compared with the experimental results in order to gain understanding of the relevant effects of the boundary conditions.
First, simple analytical second order beam theory as presented in [2] is used. Second, the solution of a fully resolved 3D model with the commercial tool ANSYS is used and third, FEM simulations with the inhouse code BACI are performed in which the simplified models for the boundary conditions are developed. The latter is a 3D parallel multiphysics simulation tool from which only the 2D structural field is used for this work. Details on our structural model can be found in [3].

2.1. Static behavior of the panel support
The setup of the panel and its support as illustrated in Fig. 1 is summarized briefly here and the reader is referred to [4, 5] for a detailed description. The elastic panel is made of spring steel (CK75) and it has a length of \( l = 300 \text{ mm} \). The width is \( b = 200 \text{ mm} \) in spanwise direction and its height is \( h = 1.47 \text{ mm} \). The elastic panel is clamped to the frame with two rows of rivets at its upstream and downstream ends. Thus, the panel has an overall length of \( 330 \text{ mm} \) of which the aforementioned \( 300 \text{ mm} \) are unsupported. The elastic part can freely move in \( z \)-direction, i.e. the height direction. In spanwise direction, the panel is unsupported and a small gap exists between the panel and frame it is riveted to. To seal the cavity underneath the elastic panel, a soft rubber foam sealing is mounted between panel and frame along the spanwise sides. An ideal fixed bearing of the panel is not possible in a real setup which has already been reported in [2]. In order to quantify this behavior, preliminary experiments were performed in which the pressure in the cavity under the panel was reduced by continuous suction such that a static deflection occurred, see [2]. Also, a comprehensive 3D model including contact between frame, rivets and panel (“3D ANSYS w/ rivets”, see Fig. 1(b)) is presented which is able to properly describe the overall real system. However, considering a coupled FSI setup, it is not desirable to model the support of the panel in such a detail and therefore, simplified models are necessary for the panel support.

The most trivial support scenario is a fully fixed clamping (“2D fixed support”) of the panel at its ends. In Fig. 2, the deflection in \( z \)-direction and the corresponding membrane strain is given over the difference pressure. The experimental result gives a significantly larger deflection than our numerical results and also beam theory for a fully clamped support (“2nd order beam theory”), see Fig. 3(a). The deviation between experiment and simulation can be explained by a maximal displacement of the frame in the experi-
In addition, the rivets might act as a source of unintended motion due to their non-rigid behavior. Due to the setup, this small deflection in $x$-direction has a large influence on the deflection of the panel in $z$-direction which is in the size of millimeters. To soften the behavior of the support and model the unintended frame deflection, a spring acting in $x$-direction, i.e. the flow direction, with a stiffness of $5 \cdot 10^6$ N/m is attached to the panel ends (see Fig. 3(b)) to meet the static deflection for different pressure levels in the cavity. The results displayed in Fig. 2(a) reveal that already a simple linear spring ("2D elastic support") is appropriate to model the "soft" behavior of the steel frame and the clamping region with the rivets. The resulting displacements of the spring are in the same range as the measured deflection of the frame in $x$-direction indicating that this model assumption seems appropriate. As a further validation, the membrane strain of the numerical schemes is compared with experimental data in Fig. 2(b). The full 3D ANSYS simulation as well as the 2D simulation with elastic support reproduce the experimental measurements very accurately whereas the fixed support overestimates the strains. In summary, it is sufficient to replace the fixed support at the ends of the panel with a linear spring model in order to obtain a proper surrogate for the comprehensive 3D model, at least in the static case. As we will see in the next section this does not hold for dynamic effects.

2.2. Dynamic behavior of the panel support

Up to now, only the static deflection of the panel is considered for fitting boundary conditions, i.e. a spring stiffness which acts in $x$-direction. The corresponding numerical results in the context of FSI simulations have already been published in [1]. As the boundary conditions also influence the eigenfrequency of the panel, a comparison between experimental results and different clamping scenarios is performed. The support including the translational spring as shown in Fig. 3(b) leads to an eigenfrequency of
91 Hz which is in very good agreement with the results in [2] for a simplified 3D model as well as with beam theory. The first eigenfrequency of a fixed-fixed beam can be computed via

\[ f_{ff} = \frac{22.4}{2\pi^2} \sqrt{\frac{EI}{m'}} = 88.7 \text{ Hz} \]  

in which \( E = 206 \text{ GPa} \) is the Young’s modulus, \( I = \frac{bh^3}{12} \) is the area moment of inertia and \( m' = \rho \cdot b \cdot h \) is the mass per unit length. In order to validate the eigenfrequencies, an experiment is performed with a reduced ambient pressure in the vacuum chamber. The pressure in the cavity below the panel as well as above the panel is reduced to a mean pressure of \( 1.3 \text{ kPa} \) in order to suppress aerodynamic effects as far as possible. The panel is initially deflected with an electromagnet which is mounted below the panel. After the electromagnet is switched off, the panel oscillates in its eigenfrequency which is displayed in Fig. 4(a). The first two oscillations are neglected because the collapsing magnetic field still acts on the panel. Due to measurements of the current which drives the electromagnet a negligible influence is reached at \( t = 0.05 \text{ s} \). The remaining part of the oscillations is used to extract the eigenfrequency which is clearly dominated by the first eigenmode. A harmonic oscillation is fitted in a least squares sense to the experimental results from \( t = 0.05 \text{ s} \) to \( t = 0.1 \text{ s} \) which gives a frequency of 75 Hz. This is clearly below the eigenfrequency of the support as shown in Fig. 3(b). Hence, the real support cannot be modeled solely with the translational spring. Numerical results with a support as shown in Fig. 3(c) which does not constrain the rotation leads to a first eigenfrequency of 40 Hz which is in good agreement with theoretical analysis of a simply supported beam reading

\[ f_{ss} = \frac{9.87}{2\pi^2} \sqrt{\frac{EI}{m'}} = 39.1 \text{ Hz} . \]  

Hence, the real support is between fully rotationally constrained and rotationally free which can be modeled with a rotational spring in the numerical setup as shown in Fig. 3(d). Inserting a rotational spring with a stiffness of \( 10.6 \text{ Nm/} \text{rad} \) at the ends of the unsupported part of the panel leads to an eigenfrequency of 75 Hz which coincides with the experimental result. Fortunately, the rotational spring influences the deflection marginally and the translational spring has hardly any effect on the eigenfrequencies. Hence, it is possible to tune the translational spring in \( x \)-direction independently of the rotational spring in order to meet the static deflection as well as the oscillatory frequency.

The experiments and considerations so far exclude aerodynamic effects because of...
the low ambient pressure. Further experiments with a mean ambient pressure of 8.7 kPa and 16 kPa reveal an increased eigenfrequency of 79 Hz and 83 Hz, respectively, as displayed in Fig. 4(b). The volume above the panel is large in extent such that no influence is expected. However, the volume enclosed in the cavity below the panel increases the eigenfrequency of the overall system due to its compressibility. The compressibility modulus is defined as

\[ K = -V \frac{dp}{dV} \quad (2.3) \]

in which \( V \) is the enclosed volume and \( p \) is the pressure in the cavity. Due to the complex setup including measuring devices in the cavity below the panel it is not possible to compute the volume exactly. Assuming an ideal gas in the cavity allows to rewrite Eq. (2.3) as

\[ K = -c_1 \frac{\Delta p}{\bar{p} \Delta V} \quad (2.4) \]

with a constant \( c_1 \), and in which the partial derivative is replaced with a linearized form based on the assumption that the change in volume is small compared to the absolute volume in the cavity. This assumption is valid because the maximum panel deflection in \( z \)-direction is in the range of millimeters which is small compared to the extent of the cavity. The pressure \( p \) in the cavity can be separated in a mean value \( \bar{p} \) and its deviation \( \Delta p \) such that \( p = \bar{p} + \Delta p \). Reordering of Eq. (2.4) leads to

\[ \Delta p = -c_2 \cdot \bar{p} \cdot \Delta V \quad (2.5) \]

which includes the constant \( c_2 \). From the experiments with the magnet release it is possible to fit \( c_2 = 27.7 \, \text{m}^3 \) which manifests as a pressure change in the cavity depending on the panel deflection as depicted in Fig. 4(c). This finally enables us to compute the pressure difference acting on the bottom of the panel depending on the mean pressure \( \bar{p} \), which can be measured, and the change in volume \( \Delta V \), which is computed numerically. Adding this modeling approach increases the eigenfrequency for a mean ambient pressure of 16 kPa from 75 Hz to 83.7 Hz which is in very good agreement with the experimental results.

In summary, several models for the boundary conditions of the structure in the FSI experiment are presented which enable us to capture the complex real support behavior. Springs acting in panel length direction at the panel’s ends surrogate the soft behavior of the steel frame and the region with the rivets. Already a linear spring is enough to meet the panel deflection in static experiments properly, but not the dynamics, i.e. the eigenfrequencies. Therefore, additional rotational springs at the panel’s ends are used to model the real support scenario in a simplified manner. Additionally, the enclosed air in the cavity below the panel influences the overall system behavior fundamentally. To avoid the task to model the cavity with the enclosed air in detail, an additional pressure contribution is computed which represents the influence from the air cushion.

3. Scalable solvers for strongly coupled problems

Having analyzed aspects of the FSI subproblem, the efficient solution of the monolithic thermo-structure interaction subproblem is investigated. We have recently presented and evaluated improved linear solvers for monolithic coupled multi-field problems as they occur in thermo-structure interaction. Special attention is given to different precon-
3.1. General purpose solvers for monolithic systems of equations

Monolithic schemes are often required to solve strongly coupled problems, when partitioned methods fail or are not able to provide a solution in a sensible amount of time [7]. However, the monolithic approach often leads to very challenging systems of linear equations: the system matrix is a big sparse matrix with a special block structure representing each of the underlying physical fields. This matrix frequently has a very bad condition number due to the distinct magnitudes of the involved physics. In real-world applications, iterative methods as GMRES [8] are used to attack this linear system, which requires efficient preconditioners for addressing the poor conditioning of the problem. Selecting a suitable preconditioner is the key point in the solution process and it is crucial to benefit from the robustness and efficiency of the monolithic schemes.

The first type of preconditioners considered in the developed framework are based on the so-called block preconditioners, see e.g. [9], which lead to efficient solvers by taking advantage of the block structure of the monolithic system. The key idea is to use approximate block inverses in order to untangle the coupling between the physical fields, and then, to use efficient algebraic multigrid (AMG) [10] solvers for the resulting uncoupled problems. The drawback of this approach is that the coupling is resolved only at the finest multigrid level. That is, using efficient AMG solvers for the underlying problems does not necessarily imply a good treatment of the coupling and a fast global solution. This drawback is overcome by Gee et al. [11] who propose an enhanced block preconditioner (referred to as monolithic AMG) for FSI applications, which enforces the coupling at all multigrid levels, and often results in a better solver performance. Monolithic AMG preconditioners are a promising approach for other coupled problems as well but, to our knowledge, this strategy has only been applied to FSI so far. We have proposed and implemented in our framework an extension of the monolithic AMG preconditioner to a generic coupled problem.

3.2. Numerical results: Rocket nozzle example

In the following, the performance of the developed solvers is shown with the thermo-mechanical simulation of a rocket nozzle. The nozzle geometry (see Fig. 5) and other problem parameters are inspired by the Vulcain rocket engine installed in the Ariane space launcher, see our paper [6] for further details.

The nozzle is equipped with 81 equidistant cooling channels which traverse the body longitudinally. Thanks to the symmetry of the problem, only a 1/81-th of the nozzle is meshed and simulated by introducing the required symmetry conditions. The computational domain is a sector of the nozzle of angle $360\degree/81$ which includes only one cooling channel, see Fig. 5 (center). The rocket nozzle is initially at rest and pre-cooled with a uniform initial temperature of $40\ K$. The thermo-mechanical loads acting on the body are the high pressure and temperature of the exhausted hot gases (namely $p_{hg}$ and $u_{hg}$) and the pressure and low temperature of the fluid filling the cooling channels (namely $p_{cc}$ and $u_{cc}$), see Fig. 5 (right). The temporal evolution of $p_{hg}$, $u_{hg}$, $p_{cc}$ and $u_{cc}$ is taken from [12] corresponding to a prototypical loading cycle for rocket nozzles. The total simulation time is $t = 5\ s$. A screen-shot of the deformation of the nozzle and the temperature field is given in Fig. 6.

After the usual space and time discretization, a non-linear problem needs to be solved at each time step, which is handled with a monolithic Newton scheme. At each Newton
iteration, the associated monolithic linear system of equations is solved with a GMRES method preconditioned with four different solvers available in the developed computational framework. The first method, namely BGS(AMG), considers an outer block Gauss-Seidel (BGS) scheme for uncoupling the fields and then independent AMG solvers are used to handle the thermal and mechanical problems separately. The second method, namely SIMPLE(AMG), is a similar method that considers an idea based on the SIMPLE method [13] for uncoupling the fields instead of BGS. The third method, namely AMG(BGS), is an extension of the monolithic AMG solver to a generic coupled problem. Finally, the fourth method, namely BGS(DD), is an outer BGS for uncoupling the fields and then standard single-level additive Schwartz preconditioners are used to attack the uncoupled problems. The fourth method is one of the most simple parallel preconditioners that can be considered in such a problem, and therefore, it is considered here as a reference.

The performance of the preconditioners is studied with a weak scalability test, see Fig. 7. In this experiment, the ratio between processors and degrees of freedom (DoF) is kept constant with a value about 12500 DoF/processor. The multigrid methods AMG(BGS), BGS(AMG) and SIMPLE(AMG) have a very good scalability as the iteration count and CPU time to solve one linear system increase only mildly with the problem size. On the other hand, the single-level method BGS(DD) is not scalable since the solver time
strongly grows as the problem size increases. The performance of the single-level method BGS(DD) is particularly poor in this complex example which demonstrates that multigrid preconditioners such as AMG(BGS), BGS(AMG) and SIMPLE(AMG) are required in this challenging setting. In conclusion, the multigrid solvers implemented in our generic computational framework are able to solve this complex example efficiently.

4. Conclusions

Different preconditioning strategies for the linear system arising from thermo-structure interaction problems are investigated. Multigrid preconditioners such as AMG(BGS), BGS(AMG) and SIMPLE(AMG) are required in this challenging setting to enable an efficient solution of the problem. All solver variants are readily applicable in the fully coupled thermo-fluid-structure interaction problem.

In the FSI subproblem, several models for the boundary conditions of the structure in the FSI experiment are presented which enable us to capture the complex real support behavior. It is obvious after the current considerations and findings that the support of the straight panel from the FSI experiment is not as trivial as expected initially.

The current considerations include small deflections around the initially undisturbed straight configuration of the panel. However, scenarios with large deflection still yield deviating results between experiments and simulation. This shows that more data is necessary from experiments to properly model the nonlinear boundary condition effects on the panel that have not yet been considered. Such experiments should include larger oscillation amplitudes to reach the nonlinear realm as well as oscillations around a mean deflection up to 5 mm which is in the range of the FSI experiments. Ideally, these experiments are performed at low ambient pressure to capture the purely structural response of the system avoiding any aerodynamic effects. Another important effect which is missing in the model is damping. The rubber foam sealing between panel and frame as well as aerodynamic damping effects need further investigation. The influence from the sealing can be analyzed with experiments at low ambient pressure such that aerodynamic damping is suppressed. For setting up a proper model for aerodynamic damping experiments at different pressure levels and large oscillation amplitudes might be necessary. Once all necessary effects on the boundary condition of the panel will be included, we expect to meet more precisely the experimental FSI results with our coupled simulation framework.
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References